Q1. Compressible fluids

The study of gases flow uncovers many interesting phenomena which have a myriad of applications, starting from boilers to airplanes and rockets. To simplify the calculations, in this problem the following assumptions will be adopted:

- The gas is ideal;
- The gas flow is steady and non-turbulent;
- The processes taking place in the flowing gas are adiabatic;
- The gas flow speed is much less than the speed of light;
- The gas flow is uniform and one-dimensional (axisymmetric);
- The effect of gravity is negligible.

The constants useful in this problem are:

- the molar mass of air, $\mu = 29.0\ \mathrm{g/mol}$;
- the ideal gas constant, $R = 8.32\ \frac{\mathrm{J}}{\mathrm{mol\cdot K}}$.

A. Bernoulli's equation **(1.5 p)**

Bernoulli's equation is the mathematical form of the law of conservation and transformation of energy for a flowing ideal gas. It bears the name of the Swiss physicist Daniel Bernoulli (1700 - 1782), who derived it in 1738. The easiest way to obtain this equation is to follow a fluid particle (a volume element of the fluid) in its way on a streamline.

A	Perform the energy balance between two points in the flowing gas, knowing the parameters (p_1, ρ_1, v_1) and (p_2, ρ_2, v_2), and derive the equation that connects these variables. The adiabatic exponent γ of the gas is also known. The parameter p is the gas pressure, ρ its density, and v its speed.	1.5 p

Solution:

The law of conservation and transformation of energy is the first law of thermodynamics. Since the processes taking place inside the gas are adiabatic, this law has the form

$$W + \Delta\mathcal{E} = 0,$$

where the work done by the gas is

$$W = \mathrm{p}_2\Delta V_2 - \mathrm{p}_1\Delta V_1 = \Delta m\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right)$$

and the variation of the total energy of the gas is

$$\Delta\mathcal{E} = \Delta\mathrm{K} + \Delta\mathrm{U} = \frac{\Delta m}{2}(v_2^2 - v_1^2) + \Delta m c_V(T_2 - T_1).$$

Hence, the energy eq. for the flow of the ideal gas is

$$\frac{v_1^2}{2} + \frac{p_1}{\rho_1} + c_V T_1 = \frac{v_2^2}{2} + \frac{p_2}{\rho_2} + c_V T_2.$$

Making use of the Robert Mayer eq.,

$$c_p - c_V = \frac{R}{\mu},$$

while

$$\frac{c_p}{c_V} = \gamma,$$

then

$$c_V = \frac{R}{\mu(\gamma - 1)}.$$

Moreover, from the Clapeyron – Mendeleev eq. it follows that

$$T = \frac{p\mu}{\rho R}.$$

Hence, the eq. of Bernoulli for an adiabatic flow becomes

$$\boxed{\frac{v_1^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1} = \frac{v_2^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2}}.$$

B. Propagation of a perturbation in a flowing gas **(4 p)**

In a layer of a macroscopically motionless gas system, if the pressure suddenly increases (by heating or rapid compression), the layer will begin to expand, compressing the adjacent layers. This pressure disturbance will be thus transmitted by contiguity as an elastic wave through the gas.

B1. Speed of the perturbation

The speed c of this wave is the speed of its wavefront (the most advanced surface, the points of which oscillate in phase and the thermodynamic parameters of which have the same value). If in the reference frame of the unperturbed gas the process of the wave propagation is nonsteady (the gas parameters in any point vary with time), in the reference frame of the wavefront the process will be steady, so the simple equations for steady state can be applied.

B1	Derive the mathematical expression for the speed c of the wavefront, taking into account that the thermodynamic parameters of the unperturbed gas are (p, ρ), while those "behind" the wavefront are $(p_1, \rho_1) = (p + \Delta p, \rho + \Delta\rho)$.	1.5 p

Solution:

In the reference frame of the undisturbed gas (laboratory frame, see Fig. 1), the speed of the gas in front of the wavefront (to the right) is $v = 0$ and the speed of the gas behind the wavefront (to the left) is $v_1 < c$.

In the reference frame moving together with the wavefront, the speed of the undisturbed gas laying in front of the wavefront the gas is $u = v - c = -c$ (the undisturbed gas runs against the wavefront) and the speed of the gas behind (to the left) the wavefront (or in the wakefield, or even upstream), which is still compressed, moves to the left with the speed $u_1 = v_1 - c < 0$.

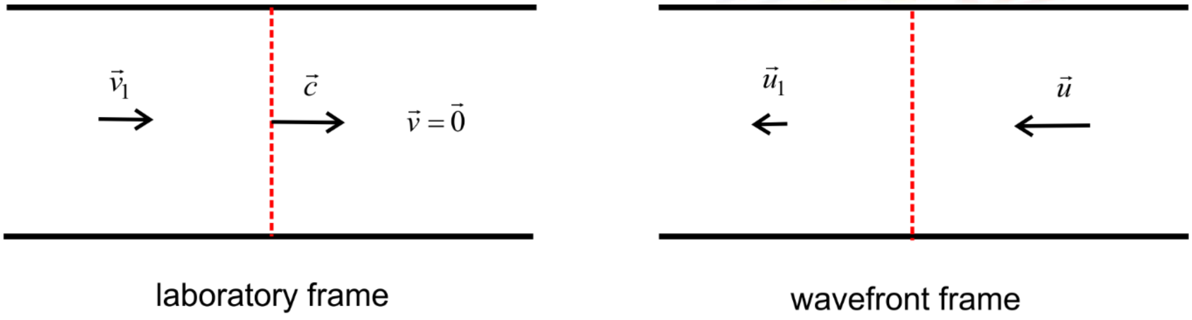


Fig.1

The equations relating the parameters of the flows in the wavefront frame are:

- The continuity equation, which expresses the mass conservation law, giving the mass flux of the flowing gas

$$\Phi = \frac{\delta m}{\delta t} = \frac{\rho S \delta x}{\delta t} = \rho S v = const.,$$

or, taking into account that the surface of the cross section is constant, for the two regions of the gas, it has the form

$$\boxed{\rho u = \rho_1 u_1.}$$

- The momentum eq., which can be written as

$$(p - p_1)S = \frac{\delta m(u_1 - u)}{\delta t} = \Phi(u_1 - u),$$

or

$$(p - p_1)S = \rho_1 S u_1^2 - \rho S u^2,$$

hence

$$\boxed{p + \rho u^2 = p_1 + \rho_1 u_1^2}.$$

Eliminating u_1 between these two eqs., the wave speed is

$$c = |u| = \sqrt{\frac{p_1 - p}{\rho\left(1 - \frac{\rho}{\rho_1}\right)}} = \sqrt{\left(1 + \frac{\Delta\rho}{\rho}\right)\frac{\Delta p}{\Delta \rho}}.$$

B.2 Sound waves

Sound waves are waves of weak disturbances ($\Delta p \ll p$ and $\Delta\rho \ll \rho$) that travel fast enough, their speed being of the order of hundreds of meters per second. Due to this, the gas compressions and rarefactions can be considered as adiabatic, the adiabatic exponent being γ.

B2	Using the result from **B1**, obtain the mathematical expression for the sound speed in the gas and, using Bernoulli's eq., derive the relation between the flow speed at a given point in the gas and the local sound speed.	0.5 p

Solution:

The sound speed in a gas is

$$c = \sqrt{\frac{\Delta p(\rho + \Delta\rho)}{\rho\Delta\rho}} \cong \sqrt{\frac{\Delta p \rho}{\rho \Delta\rho}} = \sqrt{\frac{\Delta p}{\Delta \rho}}.$$

Considering the gas compression as adiabatic ($p\rho^{-\gamma} = const.$), the sound speed will be

$$\boxed{c = \sqrt{\frac{\gamma p}{\rho}}.}$$

Thus, Bernoulli's eq. becomes

$$\boxed{\frac{v_1^2}{2} + \frac{c_1^2}{\gamma - 1} = \frac{v_2^2}{2} + \frac{c_2^2}{\gamma - 1}.}$$

B.3. Mach's number

For classifying the speed performances of bodies in a fluid (*e.g.* aircrafts), as well as the flow regimes of fluids, the Swiss aeronautical engineer Jakob Ackeret (1898 – 1981) – one of the leading authorities in the 20th century aeronautics – proposed in 1929 that the ratio of the body or of the fluid's speed v and the local sound speed c in that fluid to be called Mach's number

$$M = \frac{v}{c},$$

after the name of the great Czech (then in the Austrian empire) physicist and philosopher Ernst Mach (1838 – 1916). Primarily, the value of this adimensional quantity delimitates the incompressible from the compressible behavior of a flowing fluid, in aeronautics this limit being settled to $M = 0.3$.

B3.1	Find the relative variation of the gas density as a function of Mach's number, when its motion is slowed down to a stop, its initial velocity being v, and calculate its maximum value for a flow to be considered incompressible.	0.5 p

Solution:

According to Bernoulli's eq.

$$\frac{v^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{\gamma}{\gamma - 1}\frac{p + \Delta p}{\rho + \Delta\rho}.$$

Since

$$c = \sqrt{\frac{\Delta p}{\Delta\rho}} = \sqrt{\frac{\gamma p}{\rho}},$$

then

$$\boxed{\frac{\Delta\rho}{\rho} = \frac{M^2}{2 - M^2}.}$$

Numerically,

$$\boxed{\left(\frac{\Delta\rho}{\rho}\right)_{max} = 4{,}71 \cdot 10^{-2} = 5\%.}$$

B3.2	The pressure at the nose of an aircraft in flight was found to be $1.92\cdot10^5$ Pa and the speed of air relative to the aircraft was zero at this point. The pressure and temperature of the undisturbed air were $1.01\cdot10^5$ Pa and $21.1\ ^\circ\mathrm{C}$ respectively. The adiabatic exponent for this temperature is $\gamma = 1.40$. Find the speed and the Mach number of the aircraft.	0.5 p

Solution:

According to momentum eq., written in the reference frame of the aircraft

$$p_{stagnation} = p + \rho v^2,$$

Then, using the Clapeyron – Mendeleev eq., the result is

$$v = \sqrt{\frac{p_{stagnation} - p}{\rho}} = \sqrt{\left(\frac{p_{stagnation}}{p} - 1\right)\frac{RT}{\mu}} = 276\,\frac{\mathrm{m}}{\mathrm{s}}.$$

Since

$$c = \sqrt{\gamma\frac{RT}{\mu}} = 344\,\frac{\mathrm{m}}{\mathrm{s}},$$

then

$$M = \frac{v}{c} = 0.802\,.$$

B3.3	When a gas is flowing through a pipe, it exerts a friction force on the fluid, which is not always negligible. If at the entrance of such a pipe the static pressure in the flowing fluid is $p_1 = 6.90\cdot10^5$ Pa and the Mach number is $M_1 = 0.700$, while at the exit $M_2 = 1.00$, find the expression and the numerical value of the force with which the fluid is acting on the pipe. The adiabatic exponent is $\gamma = 1.40$, the constant cross section of the pipe is $S = 9.29\cdot10^{-2}\ \mathrm{m}^2$, and the relative increase of the gas temperature through the pipe is $5.00\cdot10^{-3}$.	1.0 p

Solution:

From the momentum eq. it follows that

$$F_f = S[(p_1 + \rho_1 v_1^2) - (p_2 + \rho_2 v_2^2)] = S[(p_1 + \rho_1 c_1^2 M_1^2) - (p_2 + \rho_2 c_2^2 M_2^2)]$$
$$= S[(p_1 + \gamma p_1 M_1^2) - (p_2 + \gamma p_2 M_2^2)] = S[p_1(1+\gamma M_1^2) - p_2(1+\gamma M_2^2)].$$

Applying the Clapeyron – Mendeleev eq. for both ends of the pipe, we get

$$\frac{p_1}{p_2} = \frac{\rho_1 T_1}{\rho_2 T_2}.$$

On the other hand, from the continuity eq.

$$\rho_1 v_1 = \rho_2 v_2$$

it follows that

$$M_1\sqrt{p_1\rho_1} = M_2\sqrt{p_2\rho_2},$$

so,

$$p_2 = \frac{M_1 p_1}{M_2}\sqrt{\frac{T_2}{T_1}}.$$

Consequently,

$$F_f = p_1 S\left[(1+\gamma M_1^2) - \frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}}(1+\gamma M_2^2)\right] = p_1 S\left[(1+\gamma M_1^2) - \frac{M_1}{M_2}\sqrt{\frac{T_1+\Delta T}{T_1}}(1+\gamma M_2^2)\right]$$

$$= p_1 S\left[(1+\gamma M_1^2) - \frac{M_1}{M_2}\sqrt{1+\frac{\Delta T}{T_1}}(1+\gamma M_2^2)\right].$$

The numerical value of this force is

$$F_f = 116\ \mathrm{N}.$$

C. Shock waves **(4.5 p)**

There are two types of acoustic waves in a gas: sound waves and shock waves. The latter appear when a body moves in a gas with a supersonic speed (*i.e.* the relative speed of the body with respect to that of the gas is greater than the sound speed). At supersonic speeds, in front of the body appears a very thin layer of gas with a higher pressure, called *compression shock*. This kind of special acoustic waves were studied by Mach, so the envelope of such a wave is known as Mach's cone, having the body in its apex. Passing through the compression shock, the thermodynamic parameters of the gas change abruptly. The Mach's cone is an example of an oblique shock, but we are interested here mainly in normal shocks, for which the shock wavefront is perpendicular on the body or fluid velocity.

For shock waves the pressure/density differences between the two sides of the wavefront can reach very high values. Passing through the wavefront, the thermodynamic parameters vary abruptly, with a sudden jump. This is another reason for which a shock wavefront is called a pressure or a compression shock.

C.1 The shock adiabat

The gas compressed by the shock wave undergoes an irreversible adiabatic process which cannot be described by Poisson's equation. However, an equation for the shock adiabat was deduced towards the end of the XIX$^{\text{th}}$ century by the Scottish physicist William Rankine (1820 – 1872) and, independently by him, by the French engineer Pierre Henri Hugoniot (1851 – 1887), using the mass and energy conservation, as well as the momentum equation. The Rankine – Hugoniot equation, or the shock adiabat, relates the pressure and the density of the gas compressed by a shock wave.

C1	Denoting with p and ρ the gas pressure and density in front of the compression shock (which are known), and with p_1 and ρ_1 the same parameters behind the shock (which are unknown), show that the pressure ratio $\frac{p_1}{p} = y_1$ is related with the density ratio $\frac{\rho_1}{\rho} = x_1$ by a relation of the form $$y_1 = \frac{\alpha x_1 - \beta}{\tau - \sigma x_1}.$$ Find the coefficients α, β, τ and σ. The adiabatic exponent γ of the gas is	1.5 p

	known. *Note: For simplicity, use a stream tube with a constant cross section, perpendicular to the wavefront of the normal shock.*	

Solution:

In the reference frame moving with the shock the processes are steady. If, in the laboratory frame, v is the velocity of the body passing through the stationary gas, which is also the speed of the compression shock, and v_1 that of the gas behind the compression shock, then in the shock reference frame, the speed of the undisturbed gas will be $u = -v$, while that of the gas behind the shock will be $u_1 = v_1 - v < 0$.

Since there are three unknown variables, u_1, p_1 and ρ_1, we need three equations to solve the problem. They are:

- The mass conservation eq. (the mass flow rate eq.), which, written for a stream tube with a constant cross section, passing perpendicularly through the normal shock, has the form

$$\rho u = \rho_1 u_1;$$

- The momentum equation

$$p + \rho u^2 = p_1 + \rho_1 u_1^2.$$

- The energy conservation equation (Bernoulli eq.)

$$\frac{u^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{u_1^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1}.$$

Using these eqs. (eliminating u_1), we find that

$$\begin{cases} \dfrac{\rho u^2}{p} = \dfrac{2\gamma}{\gamma - 1}\dfrac{x_1}{\dfrac{\gamma + 1}{\gamma - 1} - x_1} \\ \dfrac{p_1}{p} = 1 + \dfrac{\rho u^2}{p}\left(1 - \dfrac{1}{x_1}\right) \end{cases}.$$

Hence,

$$\boxed{y_1 = \frac{\dfrac{\gamma + 1}{\gamma - 1}x_1 - 1}{\dfrac{\gamma + 1}{\gamma - 1} - x_1}}.$$

This is the shock adiabat equation. Consequently, the requested coefficients are

$$\alpha = \tau = \frac{\gamma + 1}{\gamma - 1}$$

and

$$\beta = \sigma = 1.$$

C.2 A shockwave created by an explosion

An explosion creates a spherically shockwave propagating radially into still air at $p_s = 1.01 \cdot 10^5$ Pa and $t_s = 20.0$ °C. A recording instrument registers a maximum pressure of $p_1 = 1.48 \cdot 10^6$ Pa as the shock wavefront passes by. The adiabatic coefficient of air for this compression shock is $\gamma = 1.38$.

C2.1	Determine the air temperature increase $\frac{T_1}{T_s}$ under the action of the compression shock.	0.5 p

Solution:
From the shock adiabat

$$y_1 = \frac{\alpha x_1 - 1}{\alpha - x_1},$$

it follows that

$$x_1 = \frac{\alpha y_1 + 1}{\alpha + y_1},$$

where,

$$y_1 = \frac{p_1}{p_s} = 14.7$$

and

$$\alpha = \frac{\gamma + 1}{\gamma - 1} = 6.26.$$

Hence,

$$\frac{\rho_1}{\rho_s} = 4.44.$$

Since $T_s = 293$ K, the still air density is given by the Clapeyron – Mendeleev eq.,

$$\rho_s = \frac{p_s}{\frac{R}{\mu} T_s} = 1.20\ \frac{\mathrm{kg}}{\mathrm{m}^3}.$$

Under these circumstances, the air density behind the shock wavefront is $\rho_1 = 5.33\ \frac{\mathrm{kg}}{\mathrm{m}^3}$, so the temperature behind the shock is

$$T_1 = \frac{p_1 \mu}{\rho_1 R} = 968\ \mathrm{K,\ or}\ t_1 = 695\ ^\circ\mathrm{C},$$

the temperature ratio being

$$\frac{T_1}{T} = 3.30\,.$$

C2.2	Determine the Mach's number corresponding to the speed of the shockwave.	0.5 p

Solution:
From the above eqs. it follows that

$$c_{sh} = \sqrt{\frac{\rho_1}{\rho_s}\frac{p_1 - p_s}{\rho_1 - \rho_s}} = \sqrt{\frac{\rho_1}{\rho_s}\frac{\frac{p_1}{p_s} - 1}{\frac{\rho_1}{\rho_s} - 1}\frac{p}{\rho_s}} = c\sqrt{\frac{x_1}{\gamma}\frac{y_1 - 1}{x_1 - 1}},$$

or

$$M = 3.58\,.$$

C2.3	Determine the wind's speed v_1 following the shock wavefront, with respect to a fixed observer.	0.5 p

Solution:

$$v_1 = v - |u_1| = c_{sh}\left(1 - \frac{\rho_s}{\rho_1}\right) = 341 \cdot 0.775 = 264\ \left(\frac{\text{m}}{\text{s}}\right).$$

During the compression shock the gas temperature and pressure sharply increase, much more than in a quasistatic adiabatic compression. After the shock, the gas expands adiabatically, but because the slope of the adiabatic process is smaller than that of the adiabatic shock, when the gas density reaches again the initial value, its pressure p_2 is still higher than that of the unperturbed gas, p_s.

C2.4	Derive the ratios $\frac{p_2}{p_s}$ and $\frac{T_2}{T_s}$ at the end of the expansion process and calculate the numerical values of p_2 and T_2.	0.5 p

Solution:

At the end of the adiabatic expansion the requested ratios are

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \equiv \left(\frac{\rho_s}{\rho_1}\right)^{\gamma} = \left(\frac{\rho_1}{\rho_s}\right)^{-\gamma}$$

and

$$\frac{T_2}{T_1} = \frac{\frac{p_2\mu}{\rho_s R}}{\frac{p_1\mu}{\rho_1 R}} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_s}.$$

Since

$$\frac{p_1}{p_s} = \frac{\frac{\gamma+1}{\gamma-1}\frac{\rho_1}{\rho_s} - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_1}{\rho_s}},$$

then

$$\frac{p_2}{p_s} = \frac{\frac{\gamma+1}{\gamma-1}\frac{\rho_1}{\rho_s} - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_1}{\rho_s}}\left(\frac{\rho_1}{\rho_s}\right)^{-\gamma} = 1.66$$

and

$$\frac{T_2}{T_s} = \frac{\frac{p_2\mu}{\rho_s R}}{\frac{p_s\mu}{\rho_s R}} = \frac{p_2}{p_s} = 1.66\ .$$

So, $p_2 = 1.68 \cdot 10^5$ Pa and $T_2 = 486$ K ($t_2 = 213$ °C).

C2.5	From this point the gas is cooling until it reaches the initial state. Assuming that for the entire cyclic process the adiabatic exponent has the same value, derive an expression for the entropy variation of the mass unit of air during the compression shock and calculate its numerical value.	1.0 p

Solution:

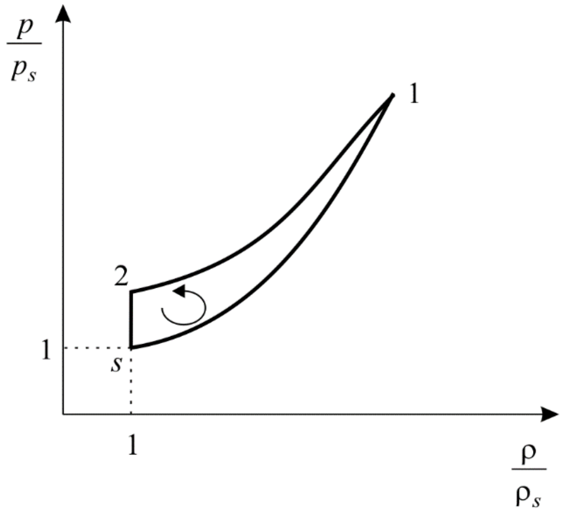


Fig. 2

Admitting that the cycle is enclosed by an isochoric process, since the entropy variation during the entire cycle, as well as for the adiabatic expansion are zero, then

$$\frac{\Delta S_{shock}}{\Delta m} = -\frac{\Delta S_V}{\Delta m} = -\frac{C_V}{\mu} ln\frac{T_s}{T_2} = \frac{1}{\gamma - 1}\frac{R}{\mu} ln\frac{T_2}{T_s} = 383 \left(\frac{\mathrm{J}}{\mathrm{kg{\cdot}K}}\right).$$

Q2. Sources, atoms, and spectra

proposed by prof. Florea ULIU, PhD, University of Craiova

A. Light source

In its proper reference frame, a point source emits light in the form of a divergent conical beam, with the angular width of 90° (from -45° to +45° with respect to the cone axis). In a reference frame which moves towards the source with an unknown speed v, the angular width of the beam is of only 60° (from -30° to +30° with respect to the same cone axis). The light speed in vacuum is $c = 2.998 \cdot 10^8\ \frac{m}{s}$.

A	Determine the speed v of the source.	2.50 p.

1st Solution:

Starting from the Lorentz transformations it is easy to obtain the so-called formula for the aberration of light

$$\tan\frac{\phi}{2} = \sqrt{\frac{1-v/c}{1+v/c}}\tan\frac{\phi'}{2},$$

so that

$$v = c\frac{\tan^2(\phi'/2) - \tan^2(\phi/2)}{\tan^2(\phi'/2) + \tan^2(\phi/2)}.$$

With $\phi = 30^\circ$ and $\phi' = 45^\circ$, it follows immediately that $v/c = 0.4098$, or $v = 1.229 \cdot 10^8 m/s$.

2nd Solution:

This solution is based on the transformation relations for velocities. Let x' and x be the axes of the cone in the proper reference frame, K' and the other one, K. The velocity of K' with respect to K is v. Consider a photon moving on the external surface of the cone with the apex angle of 90° and with x' as a symmetry axis. Then, $v_x' = c\cos 45^\circ = c/\sqrt{2}$. The other component has the same value $v_y' = c\sin 45^\circ = c/\sqrt{2}$. The speed v_x in reference frame K is given by the transformation law

$$v_x = \frac{v + v_x'}{1 + vv_x'/c^2} = \frac{v + c/\sqrt{2}}{1 + vc/c^2\sqrt{2}} = c\frac{c + v\sqrt{2}}{v + c\sqrt{2}}. \quad (*)$$

Applying the other transformation law (for the y, respectively y' components of the velocities), we will get

$$v_y = v_y'\frac{\sqrt{1-(v/c)^2}}{1 + vv_x'/c^2} = \frac{c}{\sqrt{2}}\cdot\frac{\sqrt{1-(v/c)^2}}{1 + v/c\sqrt{2}}. \quad (**)$$

Note: This expression for v_y *could be obtained from* $v_y = \sqrt{c^2 - v_x^2}$ *, with* v_x *given by eq. (*). Here it was taken into account that the photon moves with the same speed,* c*, both in* K'*, as well as in* K*.*

From $\tan 30^\circ = v_y / v_x = 1/\sqrt{3}$, using eqs. (*) and (**), it follows that $v = c\dfrac{\sqrt{3}-\sqrt{2}}{2-\sqrt{3/2}} = 0.4100c$, or

$v = 1.229 \cdot 10^8$ m/s.

<u>3rd Solution:</u>

This method uses directly the Lorentz transformations, written in differential form. So,

$$\Delta x = \gamma(\Delta x' + v\Delta t') \text{ and } \Delta y = \Delta y', \quad (***).$$

Consider a photon moving on the external surface of the cone, in the reference frame K'. If this photon correlates two events having a spatial separation $\Delta x'$ on the x' axis, their spatial separation on the y' axis is $\Delta y' = \Delta x'$ (because the moving direction of the photon makes 45° with the axis x'). The time interval separating the events is $\Delta t' = \Delta x' / c_x$, with $c_x = c/\sqrt{2}$. Thus, $\Delta t' = \Delta x' \sqrt{2}/c$. *[It could be reasoned considering that the photon moves with the speed* c *on a distance* $\Delta x' . \sqrt{2}$ *].*

Now, eq. (***) takes the form

$$\Delta x = \gamma\left(\Delta x' + v\frac{\sqrt{2}}{c}\Delta x'\right) = \gamma \Delta x'\left(1 + \frac{v\sqrt{2}}{c}\right), \text{ and } \Delta y = \Delta x', \text{ respectively.}$$

Knowing that $\tan 30^\circ = \Delta y / \Delta x = 1/\sqrt{3}$ and, using the usual notation $\beta = v/c$, the resulting eq. is $5\beta^2 + 2\sqrt{2}\beta - 2 = 0$, the physical solution of which is $\beta = 0{,}41$. So, $v = c\beta = 0.4101c$, obtaining the same result as above.

B. Balmer emission spectrum

The spectral resolving power of a spectrometer is $R = 5 \cdot 10^5$. The spectrometer is used to observe the Balmer series in the emission spectrum of the hydrogen atom (the visible domain).

Note: The possible mechanisms of broadening of the spectral lines (Lorentzian, Gaussian, etc.) will not be considered.

B.1	Express the mathematical definition of the spectral resolving power of the instrument.	0.25 p.

<u>Solution:</u>

The spectral resolving power of an optical instrument is $R \equiv \lambda / \Delta\lambda$, where $\Delta\lambda$ is the minimum value of the difference between the wavelengths of two adjacent spectral lines, λ and $\lambda + \Delta\lambda$, which can be distinctly observed with the instrument. According to the Rayleigh criterion, this spectral (chromatic) interval corresponds to the following situation: the first zero minimum beside the principal maximum of the radiation λ coincides with the principal maximum of the same order of the radiation $\lambda + \Delta\lambda$.

B.2	Determine the highest value for the principal quantum number n of the energy level for which the spectral line emitted by an atom for the transition to the level $n' = 2$ can still be distinctly resolved by the instrument, with respect with its neighbors.	2.25 p.

Solution:

The wavelengths of Balmer series (the only series in visible) are given by $1/\lambda = K\left(1/4 - 1/n^2\right)$, where K is a positive constant (the value of which is not important now) and the quantum number n can take the values 3,4,5,..... The variation $\Delta\lambda$ of the wavelength λ must be found when the quantum number n varies with one unit (*i.e.* $\Delta n = 1$). The approximation $\Delta\lambda/\Delta n \approx d\lambda/dn$ (as if n would vary continuously) can be used. The derivative of Balmer eq. gives $-\frac{1}{\lambda^2}\frac{d\lambda}{dn} = \frac{2K}{n^3}$, so $\frac{d\lambda}{dn} = -\frac{2K}{n^3}\lambda^2$. Using this result, we can tell that the ratio $\Delta\lambda/\lambda$ when $\Delta n = 1$ is given by $\frac{\Delta\lambda}{\lambda} \approx \frac{2K}{n^3}\lambda = \frac{2}{n^3}\left(1/4 - 1/n^2\right)^{-1}$. The minus sign was abandoned because we will consider only positive variations of $\Delta\lambda$.

According to the definition, the spectral resolving power of an instrument is given by $R \equiv \lambda/\Delta\lambda$. In this case $\frac{\lambda}{\Delta\lambda} = \frac{n^3}{2}\left(\frac{1}{4} - \frac{1}{n^2}\right) = \frac{n^3}{8} - \frac{n}{2}$. Asking that this value to be smaller than the value of R given in the text, it follows that $\frac{n^3}{8} - \frac{n}{2} < R$. Knowing that $R = 5 \cdot 10^5$, we can conclude that the highest value of n is pretty big. Consequently, ignoring the second term in the left hand side of the above inequality, we get $n < 158.7$. The physical solution is $n_{\lim} = 158$.

C. Absorption spectra

The energy levels of an atom are given by $E_n = -\frac{A}{n^2}$, where n is an integer and A is a positive constant. Among the adjacent spectral lines which, at room temperature, the atom can absorb, two have the wavelengths $97.5\ \text{nm}$ and $102.8\ \text{nm}$, respectively. The elementary electric charge is $e = 1.602 \cdot 10^{-19}\ \text{C}$, the speed of light in vacuum is c= $2.998 \cdot 10^8\ \frac{\text{m}}{\text{s}}$, and Planck's constant is $h = 6.626 \cdot 10^{-34}$ J·s.

C.1	Find the values of the quantum numbers n of the energy levels implied in the transitions.	3.00 p.

An absorption line corresponds to an excited state of the atom, or to a down (quantum number n_1) – up (quantum number $n_2 > n_1$) transition. For such a transition to happen it is necessary to transmit the atom an energy $W(n_1 \to n_2) = A/n_1^2 - A/n_2^2$. If it is a photoexcitation, the wavelength of the absorbed radiation has the value $\lambda(n_1 \to n_2) = (ch/A)(1/n_1^2 - 1/n_2^2)^{-1}$. The adjacent absorption lines must correspond to the transitions $n_1 \to n_2$ and $n_1 \to n_2 + 1$. The last transition needs more energy, so it will correspond to a smaller wavelength. The constant A being unknown, we will derive the ratio of those two wavelengths:

$$\frac{\lambda(n_1 \to n_2)}{\lambda(n_1 \to n_2+1} = \frac{\frac{1}{n_1^2} - \frac{1}{n_2^2}}{\frac{1}{n_1^2} - \frac{1}{(n_2+1)^2}} = \frac{102.8}{97.5} = 1.054.$$

To solve this eq. we will write it first in the form

$$\frac{0.054}{n_1^2} = \frac{1.054}{n_2^2} - \frac{1}{(n_2+1)^2}$$

and will try, progressively, the possible values $n_2 = 2,3,4,5,...$.

For n_1, the obtained results will be:

$n_2 = 2 \Rightarrow n_1 = 0.59528$, $n_2 = 3 \Rightarrow n_1 = 0.99439 \approx 1$, $n_2 = 4 \Rightarrow n_1 = 1.44463$, $n_2 = 5 \Rightarrow n_1 = 1.93769$, $n_2 = 6 \Rightarrow n_1 = 2.46743$, and so on. Analyzing these values, the conclusion is that the absorption has as a consequence the transition from level $n_1 = 1$ to the level $n_2 = 3$ (the other values found for n_1 being far from integer values).

C.2	Determine the value of the constant **A** in joule and in electron-volt.	1.50 p.

On one hand, with $\lambda(1 \to 3) = 102.8\,\text{nm}$, we get

$$A = \frac{ch}{\lambda(1 \to 3)}[1/1 - 1/9]^{-1} = 2.174 \cdot 10^{-18}\,\text{J}.$$ **1.00 p.**

From the formula containing $\lambda(1 \to 4) = 97.5\,\text{nm}$, the result will be $A = \frac{ch}{\lambda(1 \to 4)}[1/1 - 1/16]^{-1} = 2.173 \cdot 10^{-18}\,\text{J}$, which is practically the same result.

Transforming the result in electron-volt, the result reeds

$$A = 2.1735 \cdot 10^{-18} J = 2.1735 \cdot 10^{-18} / 1.602 \cdot 10^{-19} = 13.57\,\text{eV}.$$ **0.50 p.**

C.3	Identify the nature of the atom and justify the choice made.	0.50 p.

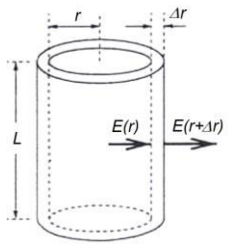
Since the ionization energy for the hydrogen atom from its fundamental state is $13.6\ \text{eV}$, the conclusion is that the atom is hydrogen.

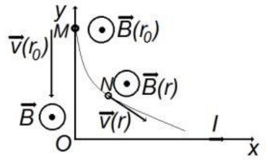
Detailed grading scheme for Theoretical Problem 3

Give appropriate grades for other way to solve the problem correctly

Theoretical Problem No. 3 (10 points)

"Squeezing" electrical charge carriers using magnetic fields

Nr. item	*Task nr. 1*		Points
1.i.	For:		**1.00p**
	expression of centrifugal force $\vec{F}_c(r) = m \cdot \omega^2 \cdot \vec{r}$	0.10p	
	expression of electrical force $\vec{F}_{el}(r) = -e \cdot \vec{E}(r)$	0.10p	
	expression of Lorentz force $\vec{F}_{elm}(r) = \pm e \cdot \vec{v} \times \vec{B}$	0.10p	
	$\vec{F}_c(r) + \vec{F}_{el}(r) + \vec{F}_{elm}(r) = 0$	0.20p	
	expression of intensity of electric field $E(r) = \left(\frac{m \cdot \omega^2}{e} \pm \omega \cdot B\right) \cdot r = \wp \cdot r$	0.10p	
			
	Gauss theorem $\left[2\pi \cdot (r + \Delta r)^2 \cdot L \cdot \wp - 2\pi \cdot (r)^2 \cdot L \cdot \wp\right] \cdot \varepsilon = 2\pi \cdot r \cdot \Delta r \cdot L \cdot \rho(r)$	0.30p	
	$\rho = \frac{2 \cdot \varepsilon \cdot \omega \cdot m}{e} \cdot \left(\omega \pm \frac{B \cdot e}{m}\right)$	0.10p	
1.ii.	For:		**0.20p**
	$\omega_0 = \frac{B \cdot e}{m}$	0.20p	
1.iii.	For:		**0.30p**
	$\omega_0 \cong 5 \times 10^7 \, rotations \cdot s^{-1}$ The situation **can not** be done in Earth's magnetic field	0.30p	
Nr. item	*Task nr. 2*		**Punctaj**
2.i.	For:		**1.00p**
	$\begin{cases} I_{S.L.} = -e \cdot n \cdot (-v_E) \cdot S \\ I_{S.L.} = e \cdot n \cdot v_E \cdot S \end{cases}$	0.10p	
	$\begin{cases} I_{S.M.} = -e \cdot n \cdot (-v_E - v_0) \cdot S + e \cdot n \cdot (-v_0) \cdot S \\ I_{S.M.} = e \cdot n \cdot v_E \cdot S \end{cases}$	0.20p	
	$I_{S.L.} = I_{S.M.}$	0.10p	

	$B_{S.L.}(r)=\frac{\mu_0 \cdot I}{2\pi \cdot r}$	0.10p	
	$B_{S.M.}(r)=\frac{\mu_0 \cdot I}{2\pi \cdot r}$	0.10p	
	$E_{wire,S.L.}(r)=0$	0.20p	
	$E_{wire,S.M.}(r)=0$	0.20p	
2.ii.	For: 		**1.50p**
	$\vec{B}_{S.L.}(r)=\frac{\mu_0 \cdot I}{2\pi \cdot r}\cdot \hat{z}$		
	expression of Lorentz force at $t=0$ $\vec{F}_{S.L.}(r_0)=e\cdot v_0 \cdot \frac{\mu_0 \cdot I}{2\pi \cdot r}\cdot \hat{x}$	0.10p	
	Lorentz force at the moment t $\vec{F}_{S.L.}(r)=e\cdot \frac{\mu_0 \cdot I}{2\pi \cdot r}\cdot (v_x \cdot \hat{y}+v_y \cdot \hat{x})$	0.30p	
	Work done by Lorentz force on the electron $dL=e\cdot \left[\left(\vec{v}\times \vec{B}_{S.L.}\right)\cdot \vec{v}\right]\cdot dt=0$ *Note: the electron kinetic energy remains constant; therefore its speed module is conserved. Velocity of electron can only change direction.*	0.50p	
	expression of initial velocity $\vec{v}_{S.M.}(r_0)=-v_0 \cdot \hat{x}-v_0 \cdot \hat{y}$	0.10p	
	expression of the velocity of electron found at the distance r to wire $\vec{v}_{S.M.}(r)=\vec{v}(r)-v_0 \cdot \hat{x}$	0.10p	
	expression of Lorentz force in SM $\vec{F}_{S.M.}(r)=-e\cdot (\vec{v}-v_0 \cdot \hat{x})\cdot \frac{\mu_0 \cdot I}{2\pi \cdot r}\cdot \hat{z}$	0.20p	
	$\vec{F}_{S.L.}(r)-\vec{F}_{S.M.}(r)=e\cdot \frac{\mu_0 \cdot I}{2\pi \cdot r}\cdot v_0 \cdot \hat{y}$	0.20p	
2.iii.	For:		**1.00p**
	In the laboratory system S.L. magnetic field produces only a change of direction of speed without any variation of its module. (The trajectory is not circular because the magnetic field induction varies with the distance from electron to wire). When the electron is in the point closest to wire, its speed on Oy direction (toward the wire) is null. At $r_0/2$ the speed of electron is parallel to wire and has the value v_0.	0.50p	
	In the mobile system S.M. the value of the initial speed of electron is $v_0 \cdot \sqrt{2}$. In this frame of reference, electron simply stops when it reaches the minimum distance.(the speed on the direction Oy must change the sign – that means must pass through the value zero; the speed on Ox direction vanishes because the speed in L.S. v_0.	0.50p	

2.iv.	For: $\vec{E}(r)=\frac{\mu_0\cdot I}{2\pi\cdot r}\cdot v_0\cdot\hat{y}$ 0.20p $U(r)=-\frac{\mu_0\cdot I}{2\pi}\cdot v_0\cdot\ln(r)+const.$ 0.30p expression of total initial energy of the electron $E_{total}(r_0)=\frac{m\cdot v^2(r_0)}{2}+e\cdot\left(-\frac{\mu_0\cdot I}{2\pi}\cdot v_0\cdot\ln(r_0)+const.\right)$ 0.20p expression of energy of the electron when it was at the minimum distance from wire $E_{total}\left(\frac{r_0}{2}\right)=0+e\cdot\left(-\frac{\mu_0\cdot I}{2\pi}\cdot v_0\cdot\ln\left(\frac{r_0}{2}\right)+const.\right)$ 0.20p energy conservation law $\frac{m\cdot v^2(r_0)}{2\cdot e}+\left(-\frac{\mu_0\cdot I}{2\pi}\cdot v_0\cdot\ln(r_0)\right)=\left(-\frac{\mu_0\cdot I}{2\pi}\cdot v_0\cdot\ln\left(\frac{r_0}{2}\right)\right)$ 0.20p $v_0=\frac{\mu_0\cdot I\cdot e}{2\pi\cdot m}\cdot\ln 2$ 0.20p $v_0\cong 0{,}243\times 10^6\,m/s$ 0.20p	**1.50p**
2.v.	For: energy conservation law $\frac{m\cdot v_0}{2\cdot e}+\left(\frac{\mu_0\cdot I}{2\pi}\cdot\ln(r_0)\right)=\left(\frac{\mu_0\cdot I}{2\pi}\cdot\ln(D)\right)$ 0.30p $D=2\cdot r_0$ 0.20p	**0.50p**
Nr. item	***Task nr. 3***	**Punctaj**
3.i.	For: Expression of the force due to pressure $\vec{F}_p=r\cdot d\varphi\cdot 1\cdot p(r)-(r+dr)\cdot d\varphi\cdot 1\cdot p(r+dr)$ $\vec{F}_p=r\cdot d\varphi\cdot 1\cdot dp$ $\vec{F}_p=r\cdot d\varphi\cdot 1\cdot dr\cdot\frac{dp}{dr}=\frac{dp}{dr}\cdot dV$ 0.20p electrical force $\vec{F}_e=0$ 0.10p Ampére law $B(r)\cdot 2\cdot\pi\cdot r=\mu\cdot j\cdot\pi\cdot r^2$ $B(r)=\frac{\mu\cdot r}{2}\cdot j$ 0.10p Total Lorentz force acting on „elementary element” of the plasma column $\left\|\vec{F}_m\right\|=e\cdot r\cdot d\varphi\cdot 1\cdot dr\cdot n(r)\cdot B(r)\cdot(\|v_i\|+\|v_e\|)$ 0.20p current density $j=e\cdot n(r)\cdot(\|v_i\|+\|v_e\|)$ 0.10p	**1.20p**

	$\left\|\vec{F}_m\right\| = \frac{\mu \cdot r}{2} \cdot j^2 \cdot dV$	0.10p	
	Condition for achieving balance of „elementary element” from the plasma column $\vec{F}_p + \vec{F}_m = 0$	0.20p	
	$\frac{dp}{dr} = -\frac{\mu \cdot r}{2} \cdot j^2$	0.20p	
3.ii.	For:		**1.00p**
	$\int_{p_0}^{p(r)} dp = -\frac{\mu \cdot j^2}{2} \cdot \int_0^r r \cdot dr$	0.20p	
	$p(r) = p_0 - \frac{\mu \cdot j^2}{4} \cdot r^2$	0.20p	
	plasma column located in vacuum $p(R) = 0$	0.20p	
	The pressure at the axis of the plasma column $p_0 = \frac{\mu \cdot j^2}{4} \cdot R^2$	0.20p	
	$p(r) = \frac{\mu \cdot j^2}{4} \cdot \left(R^2 - r^2\right)$	0.20p	
3.iii.	For:		**0,80p**
	concentration $\overline{n}$ of particles in considered volume $\overline{n} = \frac{p}{k_B \cdot T}$	0.20p	
	$N = \frac{1}{k_B \cdot T} \cdot \int_0^R 2\pi \cdot r \cdot L \cdot p(r) \cdot dr$	0.20p	
	$N = \frac{\mu}{8k_B \cdot \pi} \cdot \frac{L \cdot I^2}{T}$	0.40p	
TOTAL			**10p**

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Prof. dr. Delia DAVIDESCU
Conf. univ. dr. Adrian DAFINEI

ANSWER SHEET

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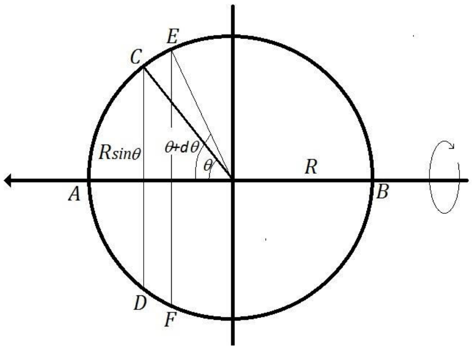
to be used as Marking Scheme

Experimental Problem No. 1 (10 points)

Secrets of toys

Task No. 1

Show that the expression of the moment of inertia of the globe when rotating around its diameter is $\frac{2}{3}\cdot m\cdot R^2$. Assume that the thickness of globe is negligible. Denote the mass of globe by m and the radius by R.



The spherical surface of Figure which is in rotation about the axis AB may be regarded as a collection of rings marked in figure by $CDFE$. Mass of this ring will be — 1.00p

$$dm = \frac{m}{4\pi\cdot R^2}\cdot dS = \frac{m}{2}\cdot \sin\theta\cdot d\theta$$

$$dJ = R^2\cdot dm = \frac{m\cdot R^2}{2}\cdot \sin^3\theta\cdot d\theta$$

The moment of inertia of the sphere, which rotates around a diameter has the expression

$$J = \frac{m\cdot R^2}{2} = \frac{2}{3}\cdot m\cdot R^2$$

Task No. 2

Find the expression of the moment of inertia of the globe when this rotates around an axis tangent to the globe.

Applying Steiner theorem the moment of inertia J_t when the sphere rotates around an tangent axis has the expression — 0.50p

$$J_t = J + m\cdot R^2 = \frac{2}{3}\cdot m\cdot R^2 + m\cdot R^2 = \frac{5}{3}\cdot m\cdot R^2$$

Task No. 3

3.i. Describe the method proposed

We study oscillation of the globe. The equation of oscillation is

$$\ddot{\theta}+\frac{3}{5}\cdot\frac{g}{R}\cdot\theta=0$$

Small oscillation of the globe around an tangent axis has Ω

$$\Omega=\sqrt{\frac{3}{5}\cdot\frac{g}{R}}$$

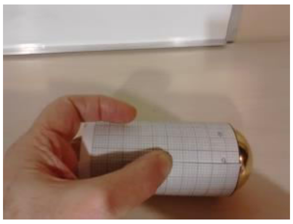
And therefore the period T

$$T=\frac{2\pi}{\Omega}=2\pi\cdot\sqrt{\frac{5\cdot R}{3\cdot g}}$$

Measuring the period of oscillation allow the determination of the length of the radius of globe in S.I.units.

1.00p

3.ii. Determine the circumference of the globe in a.u. using the format ##,# a.u.



Measuring

$$2\pi\cdot R=8{,}85\,a.u.$$

That is

$$R=1{,}41\,a.u$$

0.50p

3.iii. Fill in the appropriate box on the answer sheet a table of the data you consider as relevant.

The relevant data are those allowing determining the period of small oscillations of the globe. It can measure up to 100 oscillations of the globe. It can be fixed as the minimum necessary for maximum point, making 10 measurements of series of 30, 50 or 70 swings and determining of a weighted average value of period.

Nr	Nr. osc.	Time (s)	Perioad T (s)	Mean period $\overline{T}(s)$
1	50	23	0,460	0,460
2	50	23	0,460	
3	50	23	0,460	
4	70	32	0,457	0,457
50	30	14	0.466	0,466

$$\overline{T} = \frac{0{,}46 \times 3 + 0{,}457 + 0{,}466}{5} = 0{,}4606$$

Because

$$R = \frac{3 \cdot g}{20 \cdot \pi^2} \cdot T^2$$

The numerical value is

$$R = \frac{3 \cdot 9{,}81}{20 \cdot \pi^2} \cdot (0{,}46)^2 = 0{,}0316m \cong 3{,}1cm$$

1.50p

3.iv. Determine the value of the arbitrary unit in millimeters $1a.u. = \# mm$ and write the found value in the format ##,# (mm)

Because $R = 1{,}41a.u$, it results

$$1a.u. = \frac{3{,}1}{1{,}41} cm = 2{,}198\, cm = 21{,}9mm$$

0.50p

Task No. 4

Determine the length of cube edge and express the value in cm, using one significant figure.

Measured using a ruler marked in arbitrary units, the edge of the cube has the length of

$\ell = 2{,}5\ a.u.$

So that,

$\ell = 2{,}5 \times 21{,}9 = 54{,}75\, mm \cong 5 cm.$

0,50p

Task No. 5

5.i. Attach sheets with the observed images to the answer sheet. Give them a specific number and fill this number in the appropriate cell of data table

1.00p

5.ii. Fill a table with the measured data. Identify the measurement by noting in the appropriate cells of table the letter from the upper side of cube and the letter of side with slit of entrance of the light beam.

Nr. image	Front side (entry slit)	Upper side	Distance screen-cube a.u.	Distance central spot – first order diff. spot a.u.
1	F	C	3	2,1
2	F	U	3	2,1
3	F	B	3	2,1
4	F	E	3	2,1
5	G	C	3	3,2
6	G	U	3	3,2
7	G	B	3	3,2
8	G	E	3	3,2
9	G	C	2	1,4
10	G	C	3	2,1
11	G	C	4	2,8
12	G	C	5	3,4
13	F	C	2	1,9
14	F	C	3	2,6
15	F	C	4	3,3
16	F	C	5	4,0

1.00p

5.iii. Identify the simplest configuration of optical devices located inside the cube. Lists the devices that are inside the black box.

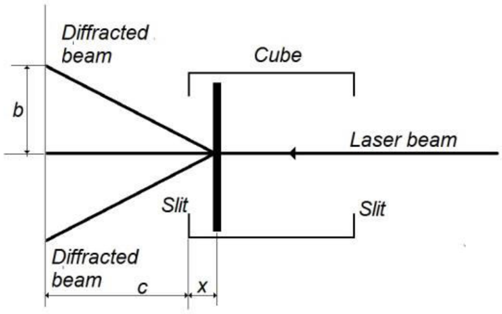
The fact that the image does not change when rotate eliminate the possibility of a prism inside. Appearance of several spots allows concluding that the box contains one or more diffraction gratings. **Inside the box there is no prism.** **There are one or two diffraction gratings perpendicular to the laser beam**	1.00p

5.iv. Using appropriate graphics, determine the distance of each optical device to the side with slit marked F. On separate papers plot the graphs you consider relevant for each device. Write down the determined values

The spots of first order diffraction are symmetrical with respect to the central axis and appear at angles determined by constant of diffraction grating d according to the relation

$d \cdot \sin\theta = \lambda$

The geometry of diffraction is



The graph of the dependence $b = b(c)$ is a line

$b = c \cdot m + n$ having the slope $m = tg\theta$ and the interception $n = x \cdot tg\theta$

Therefore the diffraction grid's distance from outlet slit is $x = \dfrac{n}{m}$

From the slope of graph can deduct the $\sin\theta = \dfrac{tg\theta}{\sqrt{1+tg^2\theta}} = \dfrac{m}{\sqrt{1+m^2}}$

Constant of the diffraction grid has the expression

$$d = \frac{\lambda}{\sin\theta} = \frac{\lambda \cdot \sqrt{1+m^2}}{m}$$

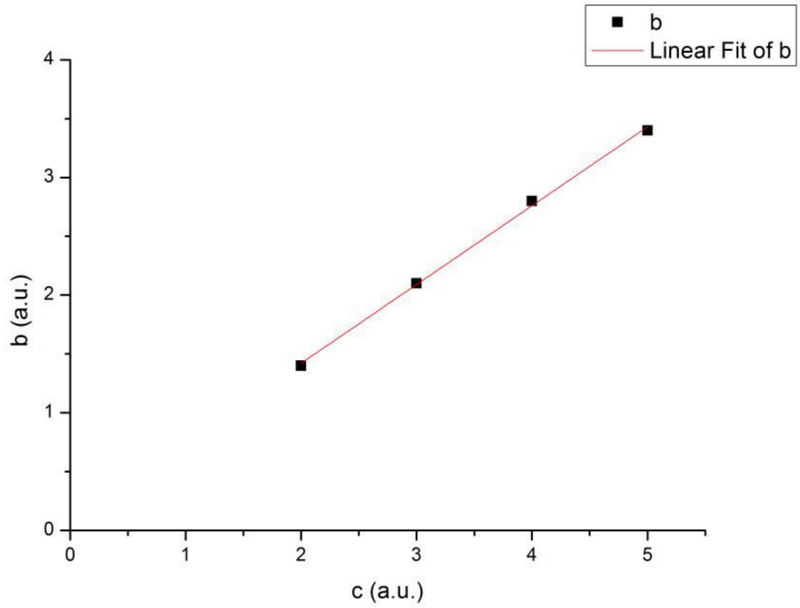
Correspondingly the number N, of lines on pe milimetru of diffraction gri dis

$$N = \frac{m}{\lambda \cdot \sqrt{1+m^2}}$$

1.00p

5.iv.

Using data sets having numbers 9-12 is plotted the following graph



From the graph,

$$\begin{cases} n = 0{,}08 \\ m = 0{,}67 \end{cases} \qquad (1)$$

1.00p

It follows that the light that comes out through the slit of side F was diffracted by a diffraction grid situated apart of side F at the distance x_F

$$x_F = \frac{n}{m} = \frac{0{,}08}{0{,}67} = 0{,}11 a.u. \qquad (2)$$

$$x_F = 0{,}11 a.u. \times 21{,}9 mm = 2{,}4 mm$$

It can accept as fair result any value between 0 and 5 mm.

Using data sets having numbers 13-16 is plotted the following graph

From the graph,

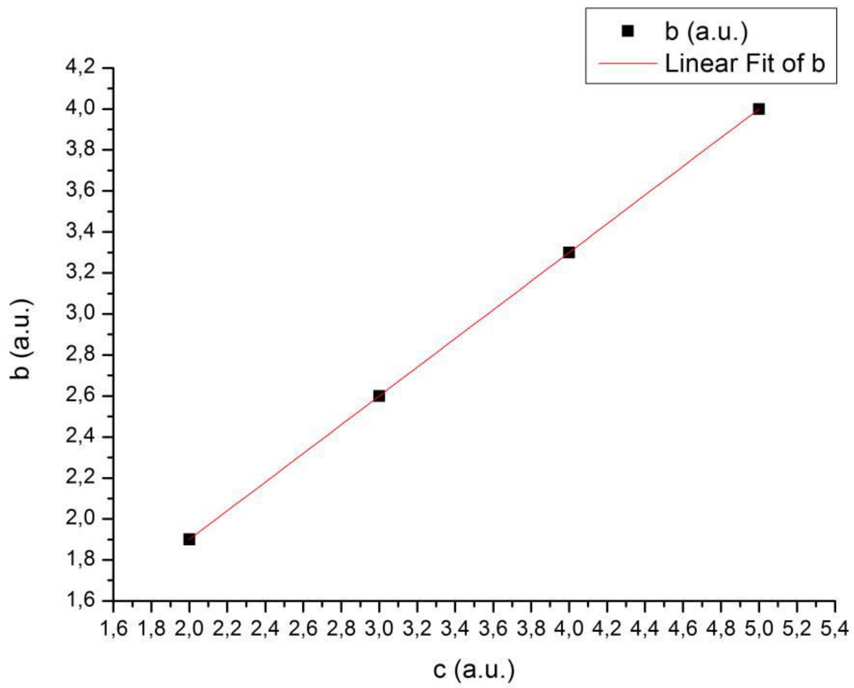
$$\begin{cases} n = 0{,}5 \\ m = 0{,}7 \end{cases} \qquad (3)$$

It follows that the light that comes out through the slit of side G was diffracted by a diffraction grid situated apart of side G at the distance x_G,

$$x_G = \frac{n}{m} = \frac{0{,}5}{0{,}7} = 0{,}71 a.u. \qquad (4)$$

$$x_G = 0{,}71 a.u. \times 21{,}9 mm = 15{,}5 mm \qquad (5)$$

5.iv.



Correspondingly the distance on side F, d_F is

$d_F = \ell - x_G = 5cm - 1{,}55cm = 3{,}45cm\,(6)$

It can accept as fair result any value of x_G between 14 and16 mm.

1.00p

5.v. Determine other geometrical features of each optical device.

Inside the cube it exists two diffraction gratings situate at calculated distances .
The grids are perpendicular on the direction of laser beam – because the images are symmetric.
The number of lines of the grid observed when the side F is illuminated is

$$N_F = \frac{m}{\lambda \cdot \sqrt{1+m^2}} = \frac{0{,}67}{630 \times 10^{-6} mm \cdot \sqrt{1+0{,}67^2}} \cong 900 tras/mm$$

Accept any results between 800 and1200 lines on mm.
The number of lines of the grid observed when the side G is illuminated is

$$N = \frac{m}{\lambda \cdot \sqrt{1+m^2}} = \frac{0{,}7}{630 \times 10^{-6} mm \cdot \sqrt{1+0{,}7^2}} \cong 900 tras/mm$$

Accept any results between 800 and1200 lines on mm.

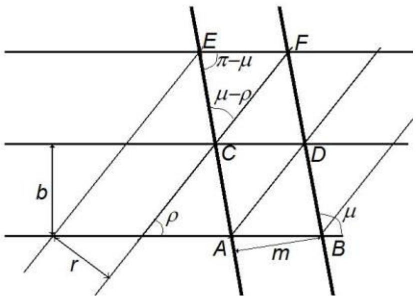
0.50p

Detailed grading scheme for Experimental Problem 2

Give appropriate grades for other way to solve the problem correctly

Experimental Problem Nr.2 - The study of Moiré patern

Nr. item	Task nr. 1 – Parallel grids		Points
1a.	For: the observation that Moiré pattern is not observed when Revelator overlaps Grid 1, the two grids having parallel lines.	0,10p	0,40p
	period of Grid 1 is identical to the period of Revelator $b = r$	0,20p	
	$b_{G1} = 0{,}76\,u.a.$	0,10p	
1b.	For: period of Moiré fringes $m = 23\,u.a.$	0,50p	0,50p
1c.	For: $m = b \cdot r / \lvert b - r \rvert$	0,10p	0,50p
	expression of the period of Base $b = \dfrac{1}{(1/r) \pm (1/m)}$	0,20p	
	values of the periods of the optical grid Grid 2 $\begin{cases} b_{G2} = 0{,}73\,u.a. \\ b'_{G2} = 0{,}79\,u.a. \end{cases}$	0,20p	

Nr. item	Task nr. 2 – Tilted grids		Points
2a.	For:  $\begin{cases} CF \cdot \sin\rho = b \\ EF \cdot \sin\rho = r \\ EF \cdot \sin\mu = m \\ CE \cdot \sin\mu = b \end{cases}$	0,40p	0,80p
	expression of the period of the grid of Moiré fringes $m = r \cdot \dfrac{\sin\mu}{\sin\rho}$	0,10p	
	The sin theorem application in ΔEFC $\dfrac{EF}{\sin(\mu-\rho)} = \dfrac{FC}{\sin\mu} = \dfrac{EC}{\sin\rho}$	0,10p	
	$\begin{cases} tg\mu = \dfrac{b \cdot \sin\rho}{b \cdot \cos\rho - r} \\ \mu = arctg\left(\dfrac{b \cdot \sin\rho}{b \cdot \cos\rho - r}\right) \end{cases}$	0,20p	

2b. For: **0,50p**

$$ctg^2\mu = \left(\frac{b\cdot\cos\rho - r}{b\cdot\sin\rho}\right)^2 = \left(\frac{1}{\sin^2\mu} - 1\right)$$ 0,20p

$$\begin{cases} \left(\dfrac{b\cdot\cos\rho - r}{b\cdot\sin\rho}\right)^2 = \left(\dfrac{r^2}{m^2\cdot\sin^2\rho} - 1\right) \\ m^2 = \dfrac{r^2\cdot b^2}{(b\cdot\cos\rho - r)^2 + b^2\cdot\sin^2\rho} \end{cases}$$ 0,20p

Expression of the period m of Moiré pattern as function of r, b and ρ

$$m = \frac{r\cdot b}{\sqrt{b^2 + r^2 - 2b\cdot r\cdot\cos\rho}}$$ 0,10p

2c. For: **1,00p**

Nr. crt.	ρ	m	Nr. crt.	ρ	m
1	14,5	3,1	15	7,5	5,7
2	14,0	3,2	16	7,0	6,1
3	13,5	3,3	17	6,5	6,5
4	13,0	3,4	18	6,0	7.0
5	12,5	3,5	19	5,5	7,7
6	12,0	3,7	20	5,0	8,6
7	11,5	3,8	21	4,5	9,6
8	11,0	3,9	22	4,0	10,6
9	10,5	4,1	23	3,5	12,0
10	10,0	4,3	24	3,0	13,6
11	9,5	4,5	25	2,5	16,0
12	9,0	4,8	26	2,0	20,0
13	8,5	5,1	27	1,5	26,0
14	8,0	5,4	28	1,0	40,0

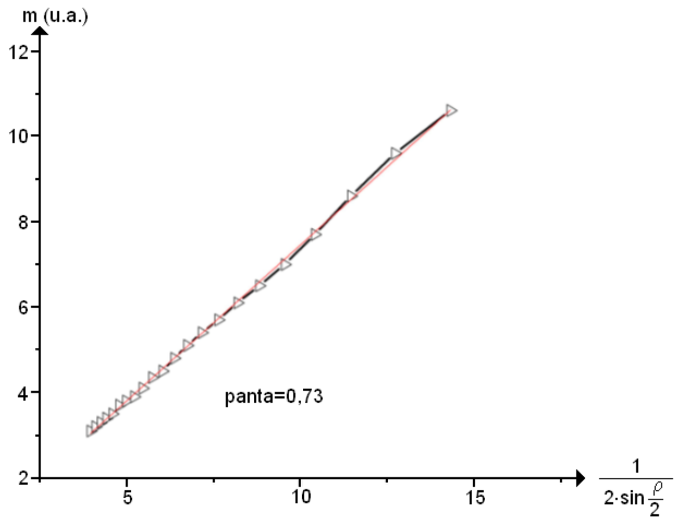
1,00p

0,1px10=1p

2d. For: **1,00p**

$m = r\cdot\dfrac{1}{2\sin(\rho/2)}$, for $r = b$ 0,20p

Eliminating data points corresponding to small angles there where measurements are imprecise; drawing of linearized dependence $m = f(1/(2\sin(\rho/2)))$, for angles $4° < \rho < 15°$ 0,10p



Slope=0,73

0,70p

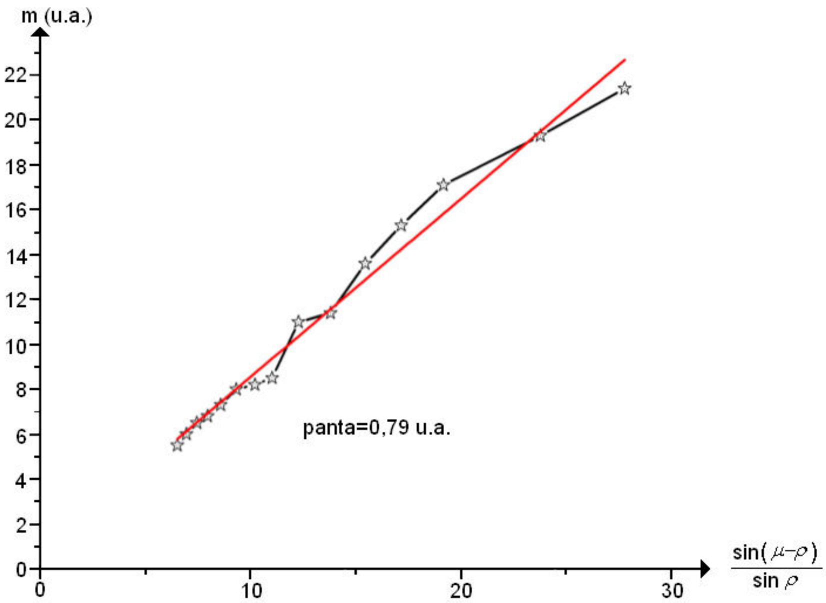
2e.	For: specification that r is the slope of graph $m=f\left(1/\left(2\sin\left(\rho/2\right)\right)\right)$ $r=0{,}73\,u.a.$ value of the period of Grid 1, determined using graph: $b'_{G1}=0{,}73\,u.a.$	 0,20p 0,20p 0,10p	**0,50p**
2f.	For: (table below) 0,1px10=1p	1,00p	**1,00p**

Nr. crt.	ρ	μ	m
1	1,0	30,0	21,4
2	1,5	40,0	19,3
3	2,0	44,0	17,1
4	2,5	51,0	15,3
5	3,0	57,0	13,6
6	3,5	61,0	11,4
7	4,0	63,0	11,0
8	4,5	64,5	8,5
9	5,0	68,0	8,2
10	5,5	69,0	8,0
11	6,0	70,0	7,3
12	6,5	71,0	6,8
13	7,0	72,5	6,5
14	7,5	73,0	6,0
15	8,0	73,5	5,5

2g. For: **1,00p**

$m=b\dfrac{\sin(\mu-\rho)}{\sin\rho}$ 0,20p

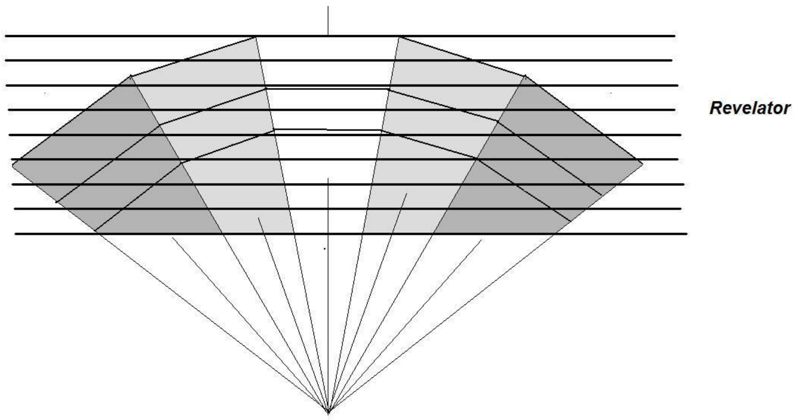
Liniarized dependence: $m=f\left(\sin(\mu-\rho)/\sin(\rho)\right)$ 0,10p



0,70p

2h.	For: specification that the slope of graph $m=f\left(\sin(\mu-\rho)/\sin(\rho)\right)$ is $tg\,\alpha'=b$ $b=0{,}79u.a.$ The value of the period of the Grid 2, determined from the graphical representation: $b'_{G2}=0{,}79\ u.a$	 0,20p 0,20p 0,10p	**0,50p**

Nr. item	*Task nr. 3 – Experimental errors of used methods*	Points
3a.	For: error on length measurements: $0{,}5\,u.a.$ 0,10p error on angles measurements: $0{,}25°$ 0,10p	**0,20p**
3b.	For: Error on measurement of period of Base: $\Delta b = \Delta m \cdot \left(\frac{r}{m+r} + \frac{mr}{(m+r)^2} \right)$ 0,10p $\Delta b = \Delta m \cdot \frac{2r}{m}$, for $m >> r$ 0,10p $b = \frac{m \cdot sin\rho}{sin(\mu - \rho)}$, for measurements on tilted grids 0,10p $b \cong \frac{m \cdot \rho}{\sin(\mu)}$, if one takes into account that the angle ρ is very small 0,10p error on the measurement of period for tilted grids: $\frac{\Delta b}{b} = \frac{\Delta m}{m} + \frac{\Delta \rho}{\rho} + \frac{\Delta \mu}{tg\mu}$ 0,20p	**0,60p**
3c.	For: error on length measurements: $\Delta m = 0{,}5\,u.a.$;error on determination of the period of grid on parallel shift: $\Delta b = 0{,}1\ u.a.$; relative error for the period of Grid 2 $\left(\frac{\Delta b}{b}\right)_{G2} = 0{,}12$ 0,20p Relative error for the tenth experimental point for Grid 2 $\left(\frac{\Delta b}{b}\right)_{G2} = 0{,}19$ 0,20p	**0,40p**

Nr. item	*Task nr. 4 - Moiré pattern using a circular grid*	Punctaj
4a.	For: Specifying that the network can be regarded as a network of regular polygons with huge number of side 0,20p  0,10p Moiré fringe width for the central area of the image $m = 6{,}1\ u.a.$ 0,20p "the period" of circular grid $\begin{cases} b_{C1} = 0{,}67\,u.a. \\ b_{C2} = 0{,}87\,u.a. \end{cases}$ 0,20p *Note: The direct observation of the networks make plausible the value* $b_{C2} = 0{,}87\,u.a.$	**0,70p**

4b.	For: specifying that the left and right areas are symmetrical and inclination of regular polygon sides (approximating circle) is identical to those areas specification of the fact that if the observed point is farther from the central zone , the pattern comes from networks stronger inclined to one another. explanation regarding the fact that the fringes are more narrow when the observed zone is farther from central diameter $m = b\dfrac{sin(\mu - \rho)}{sin\rho}$	 0,10p 0,20p 0,10p	**0,40p**
TOTAL Experimental Problem nr. 2			**10p**

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